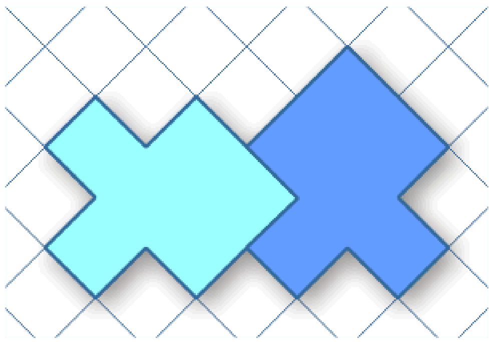
Logic GC Round 1 solutions

October 15, 2019

Ans 1:



The hint wasn't a red herring. The line isn't straight.

Ans 2:
The answer is half a hen and a half hen; that is, one hen. If one and a half hens lay one and a half eggs in one and a half days, one hen will lay one egg in one and a half days. And a hen who lays better by half will lay one and a half eggs in one and a half days, or one egg per day. So she will lay ten and a half (half a score and a half) in ten and a half days (a week and a half).

Ans 3:

Long Word

From [3], the number of common letters is at most four. From [1] and QUEST in [3], the number of common letters is at least one—S or T. From [1], from FACE and QUEST in [3], and from the fact that Q cannot occur in the thirteen-letter word without U:

	If the number of common letters is	then each of these loops contains a common letter	then each of these loops contains a common letter
i)	4	ⓆⓊⒺ [S T] C F K [T S]	F A C Ⓔ Ⓤ V Ⓠ K
ii)	1	Q U E [S T] ⒸⒻⓀ [T S]	Ⓕ A Ⓒ E U V Q Ⓚ
iii)	2	Q Ⓤ E [S T] Ⓒ F Ⓚ [T S]	F A Ⓒ E Ⓤ V Q Ⓚ
		or	
iv)	2	Q U Ⓔ [S T] ⒸⒻ K [T S]	Ⓕ A Ⓒ Ⓔ U V Q K
v)	3	ⓆⓊ E [S T] C F Ⓚ [T S]	F A C E Ⓤ V Ⓠ Ⓚ
		or	
vi)	3	Q Ⓤ Ⓔ [S T] Ⓒ F K [T S]	F A Ⓒ Ⓔ Ⓤ V Q K

A contradiction occurs in (i), (ii), (iv), and (v), so these are eliminated.

Then, from (iii) and (vi), A is a common letter.

Then, from [1] and from QUICK in [3]:

	If each of these loops contains a common letter	then each of these loops contains a common letter	Possible?
iii)	Q Ⓤ E [S T] F Ⓐ Ⓒ E Ⓒ F Ⓚ [T S] Ⓤ V Q Ⓚ	Q Ⓤ I Ⓒ Ⓚ Ⓒ F O Q E	no
vi)	Q Ⓤ Ⓔ [S T] F Ⓐ Ⓒ Ⓔ Ⓒ F K [T S] Ⓤ V Q K	Q Ⓤ I Ⓒ K Ⓒ F O Q Ⓔ	yes

Then I is the third common letter in QUICK.

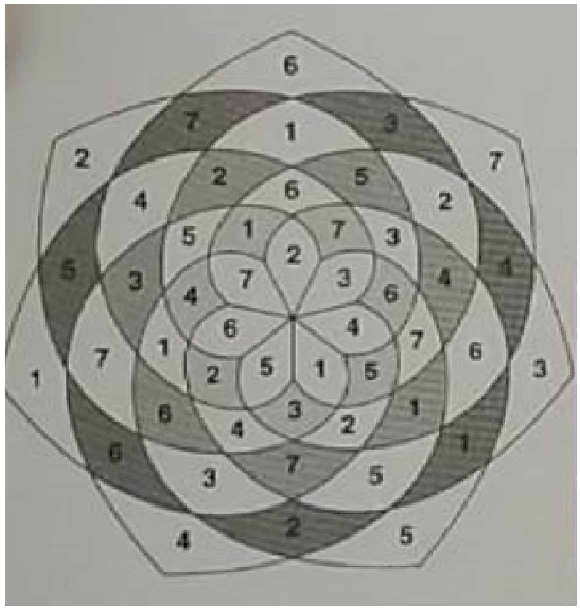
Then, from [1] and [3], the common letters in SWITCH are I, C, and S/T. Then, from [1] and [3], the common letters in WORLD are not W and not O (because I is common); so the common letters in WORLD are R, L, and D.

Ten of the thirteen letters are now known: C, U, E, A, I, R, L, D, P (not H), and B (not W). So, from [1] and [2], twenty-four letters can be arranged thus (the pairs S/T and Y/Z can only go where indicated):

U P R E D I C S A B L Y
F H J K M O Q T V W X Z

It is now easy to see that: the letters in the pair G/N should be reversed and placed after the pair U/F, the letters in the pair S/T should be reversed, and *the word is UNPREDICTABLY.*

Ans 4:



Ans 5:

Answer: 3

Solution: In B+B, the maximum carry over can be 1. For A+1 to be a two digit number(EF), A=9. Then, E=1, F=0. For 2+2 and 7+7 to give different ones digits, 1 should be added to one of them such that 2+2+1=5 and 7+7=14, or 2+2=4 and 7+7+1=15. Similarly for 3+3 and 8+8. For B+B to carry over to A, B=5,6,7,8. But, B is not equal to 5 because then G=0, but F=0. So, B=6,7,8.

If B=6:

G=2; C,D,H,I=3,4,5,7,8. D can't be 3 as then I=6, but B=6. If C=3, then D=7,8(carry over required). But for D=8, I=6:wrong and for D=7, H=7:wrong. I can't be 3 as it has to be an even number. If H=3, then C+C is either 2 or 12, so, C is either 1 or 6:wrong. Since none of the letters can be 3, therefore, 3 is absent.

If B=7:

G=4; C,D,H,I=2,3,5,6,8. D can't be 2 as then I=4, but G=4. If D=3, then I=6, C,H=2,5,8. This is not possible for any combination of C and H as C+C has H in its unit place and there is no carry over from D+D(since D=3). so, D is not 3. If C=3, then H=6, D,I=2,5,8. This is again not possible for any combination of D and I. So, C is not 3. I can't be 3 as it has to be an even number. If H=3, then, C=2 or 6. If C=2, then D,I=5,6,8:not possible. If C=6, then D,I=2,5,8:not possible. Since none of the letters can be 3, therefore, 3 is absent.

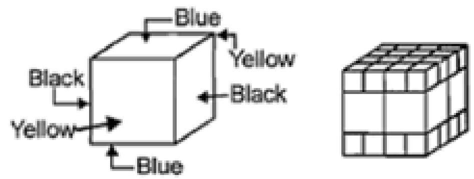
If B=8:

G=6; C,D,H,I=2,3,4,5,7. D can't be 3 as then I=6, but G=6. If C=3, then D=5,7(carry over required). If D=5, then I=0:wrong. If D=7, then I=4, H=7:wrong. So, C can't be 3. I can't be 3 as it has to be an even number. If H=3, then C+C is either 2 or 12, so, C is either 1 or 6:wrong. Since none of the letters can be 3, therefore, 3 is absent.

Therefore, the missing digit=3.

Ans 6:

Solution



In the given statement, there are 36 cubes, 32 of which are of the same size and 4 other sure of bigger size. Clearly, each side of bigger cubes is twice as large as that of similar cubes. Also, since no face of any of the larger cubes is painted blue, so each one of the larger cubes has one face painted red, one is black and all other faces are unpainted.

Leaving the 4 central cubes on each of the blue surface and 8 cubes at the edges of the block, all the rest have two faces painted.

Thus, the cubes having only two faces painted are 8 cubes on each of the blue surfaces and 4 larger cubes i.e. there are $(8 \times 2) + 4 = 20$ such cubes.

Ans 7:
Best is to shoot in the air.
If you shoot Bill and hit by chance you are surely dead.
If you shoot on Kid and hit there is a 50% chance you die before you have a chance to shoot on Bill.
If you shoot into the air Bill will shoot on Kid (he knows he is a better shooter) if he misses Bill is dead if he hits Kid dies and now you can shoot and you have 1/3 chance to win.
If you hit any of your opponents you are always in a worse situation.

Ans 8:
Answer=10
Explanation:
By symmetry, we take A is the maximum number of the three alphabets.
Let us say A can take a maximum value of 6.
Then 6! = 720 but as have we taken maximum value is 6 720 is not possible
So A should take 5. Then 5! = 120
Now from the above we know that one the B and C should take 1 as their value as 120 consists of 1.
5! + 1! = 121
If we take 4 as one of the numbers then 5!+ 1! + 4! = 145 or 1! + 4! + 5! = 145
Sum = 1 + 4 + 5 = 10.

Ans 9:
Let's number the colours red = 0, green = 1 and blue = 2. It turns out that the rule for obtaining the colour of the box z underneath boxes x and y can be expressed quite simply as:
z = 2(x + y) (mod 3)
Here, we are working in modular arithmetic to the base 3; in other words, we only care about the remainders base 3 when doing arithmetic operations. So 1 + 2 = 0, 2 + 2 = 1, 2 * 2 = 1, and so on.
To verify that the equation encapsulates the rule, we can just check all possible cases:

$0, 0- > 2(0+0) = 0(mod3)$
$1, 1- > 2(1+1) = 1(mod3)$
$2, 2- > 2(2+2) = 2(mod3)$
$0, 1- > 2(0+1) = 2(mod3)$
$0, 2- > 2(0+2) = 1(mod3)$
$1, 2- > 2(1+2) = 0(mod3)$
Now consider a size 4 triangle:

```
a b c d
 e f g
  h i
   j
```

Applying the rule, we have:
e = 2(a+b)
f = 2(b+c)
g = 2(c+d)
h = 2(e+f) = 2(2a+2b)+2(2b+2c) = a+2b+c
i = 2(f+g) = 2(2b+2c)+2(2c+2d) = b+2c+d
j = 2(h+i) = 2(a+2b+c) + 2(b+2c+d) = 2a+2d = 2(a+d)
So the value at the bottom depends only on the two values at the ends of the top row! Furthermore, the rule is exactly the same as for obtaining the colour of a box from its two immediate parents.
Now let's consider the size 10 triangle (irrelevant values are marked by periods):

```
p . . q . . r . . s
 . . . . . . . . .
  . . . . . . . .
   t . . u . . v
    . . . . . .
     . . . . .
      w . . x
       . . .
        . .
         y
```

The periods are irrelevant, since we've already shown that t depends only on p and q, u depends only on q and r, and so on. Using the rule for the size 4 triangle:
t = 2(p+q)
u = 2(q+r)
v = 2(r+s)
w = p + 2q + r
x = q + 2r + s
y = 2p + 2s = 2(p+s)
So again, y depends only on the values of p and s, and it's exactly the same rule as for computing the value directly below two adjacent entries in a row. So given any starting row of 10 colours: If the colours of the first and last boxes are the same, the box at the bottom will have the same colour; if the colours of the boxes are different, the box at the bottom will have the third colour.

Ans 10:
Looking at the question, there are only three possible combinations in which they can finish: (Y,X,Z), (Y,Z,X) or (Z,Y,X). Consider the second one. Z can't go to second position in an even number of permutations, hence this is not possible. Similarly for the third one, X can't go to the third position in odd number of permutations. Hence the answer is (Y,X,Z). Note that there doesn't exist a unique way in which the following configuration can be achieved.

Ans 11:
Jan sends Maria a box with the ring in it and one of his padlocks on it. Upon receipt Maria affixes her own padlock to box and mails it back with both padlocks on it. When Jan gets it he removes his padlock and sends the box back to Maria.

Ans 12:
The trajectory of the submarine is determined by its initial position (call it p), and its speed in units per hour (call it s). Both p and s are integers, so the set of all possible trajectories for the submarine is given by the set of all pairs (p, s) of integers.
Now, order the pairs (p, s). Does the name Cantor ring a bell? One way is to list all pairs with $|p| + |s| = 0$, then all pairs with $|p| + |s| = 1$, then all pairs with $|p| + |s| = 2$, and so on. Here's the first dozen pairs in the list, with the column t just showing the position of each pair in the list.

t	p	s
0	0	0
1	1	0
2	0	1
3	-1	0
4	0	-1
5	2	0
6	1	1
7	0	2
8	-1	1
9	-2	0
10	-1	-1
11	0	-2

The strategy is to try for the submarine in row t at time t. We look up p and s for time t, and fire at the integer p+st.
For example, when t = 11 hours, we look up (p,s) for t=6, and find (0,-2), and so we fire at 0-2*11 = -22. If the submarine had trajectory (0,-2), then this shot will sink it.
It's easy to see that all possible trajectories for the submarine appear somewhere in the list, hence this strategy will indeed sink the submarine.

Ans 13:

Randomly searching will not guarantee you find the turkey. Neither will checking every box, nor will checking the same box over and over. To find this bird, we are going to have to make some assumptions.

First, for the sake of argument, let's assume the turkey is in an even-numbered box, meaning either box 2 or box 4.

Let's say you check box 2. If you find the turkey, all is well and good in the world and Thanksgiving can proceed. If not, then you know the turkey must have been in box 4 (again, this is based on an initial assumption that the turkey was in an even-numbered box).

If the turkey was in box 4 on the first day, when you checked box 2, then it must move to either box 3 or box 5 on the second day. So on the second day, check box 3. If the turkey is there, you win. If not, it must be in box 5, and if the turkey is in box 5 on the second day, it must move to box 4 on the third day, and so you check box 4 on the third day and find the turkey.

Now, the above scenario—checking box 2, 3, and then 4—will always let you win assuming the turkey started in an even-numbered box. But, of course, that might not be the case. Now let's look at the scenario if the turkey started in an odd-numbered box—1, 3, or 5.

If the turkey is in box 1, 3, or 5, then on the second day, it must have moved to either box 2 or 4. On the third day, it must have moved back to box 1, 3, or 5. And on the fourth, the turkey again must have moved to either box 2 or 4.

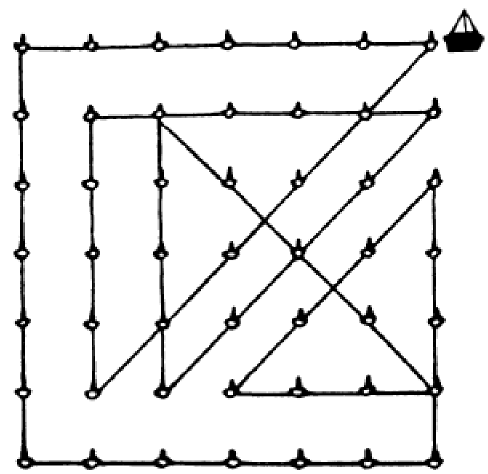
You can probably sense we've discovered something important here: If the turkey started in an odd-numbered box, then after checking for three days, it must be in an even-numbered box. In other words, if the turkey started in an odd-numbered box, at the start of the fourth day, it must be in an even-numbered box. We now must combine the two scenarios.

First, we know from the first example that if you check box 2 and then 3 and then 4, you will find the turkey if it started in an even-numbered box. Let's say you check 2, 3, and 4 on the first three days, and you do not find the turkey. That means it must have started in an odd-numbered box, which also means that on the start of the fourth day, it must be in an even-numbered box. So, on the fourth day, if you have not found the turkey, you repeat the process, because you know that now it must be in an even box.

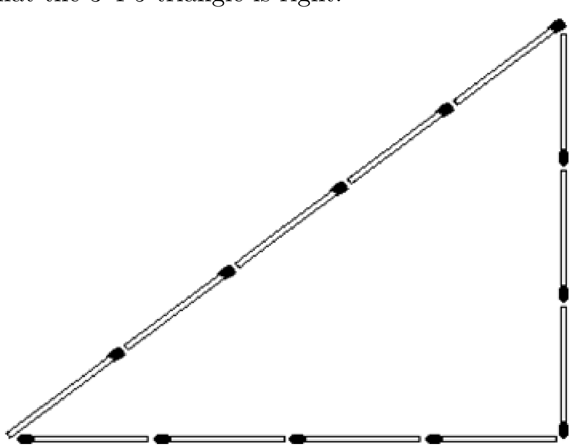
So here is the solution: Check box 2 on the first day, then 3 on the second day, and then 4 on the third day. If the turkey was in an even box, you are guaranteed to find it on one of those first three days. If you don't find it, then it must have started in an odd-numbered box, and on the start of the fourth day, it must be in an even-numbered box. So you then check box 2 on the fourth day, then box 3 on the fifth day, and finally

box 4 on the sixth day. No matter what, you will have found the turkey.
In short: check box 2 then 3 then 4, and if you do not find the turkey, check box 2 then 3 then 4 again.

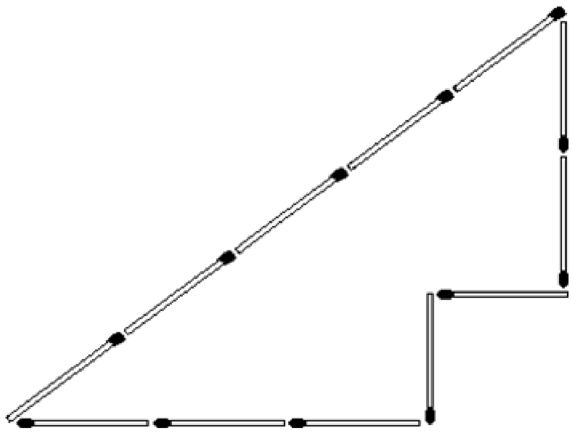
Ans 14:



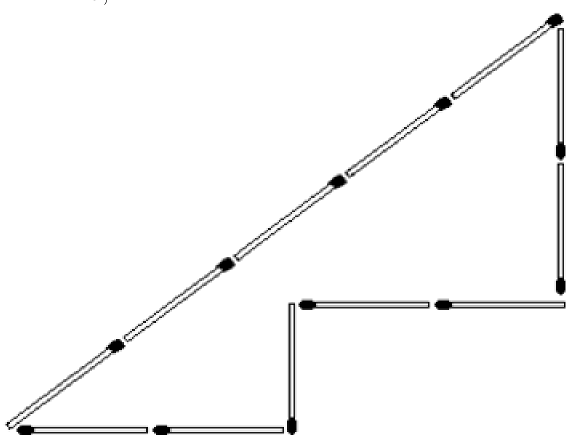
Ans 15:
Among many triangles with perimeter 12, the most famous is the 3-4-5 triangle, the triangle with sides 3, 4, and 5 (or a similar one.) Since $3^2 + 4^2 = 5^2$, we are assured by the converse of the Pythagorean theorem that the 3-4-5 triangle is right.



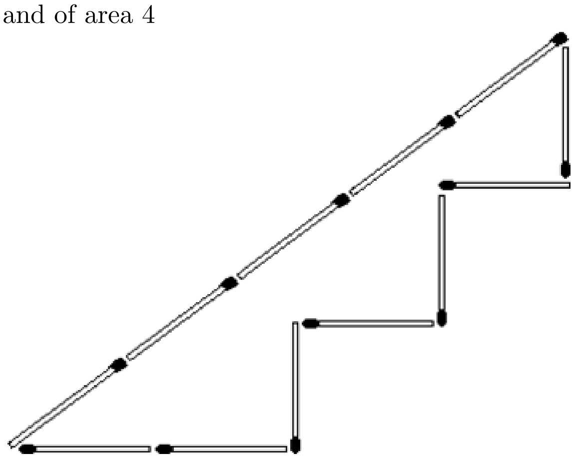
The right triangle with sides 3 and 4 has area of $6 = 3{\cdot}4/2$. By removing a "unit square" at a time we get consecutively shapes



of area 5,



and of area 4



and finally one of area 3, too.

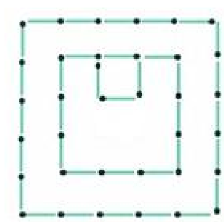
Ans 16:
Let's start with JKL = 9 * LQ. Note that L appear on both the side. Also, after multiplying LQ by 9 the answer should have L at the unit's place. The possible values of LQ are 19, 28, 37, 46, 55, 64, 73, 82 and 91; out of which only 64, 73 and 82 satisfies the condition. (As all alphabets should represent different digits)
Now, consider PQR = 6 * LQ. Out of three short-listed values, only 73 satisfies the equation.
Also, ZYX = 3 * LQ is satisfied by 73.

Hence, Z=2, Y=1, X=9, P=4, Q=3, R=8, J=6, K=5, L=7
$219/3 = 438/6 = 657/9 = 73$

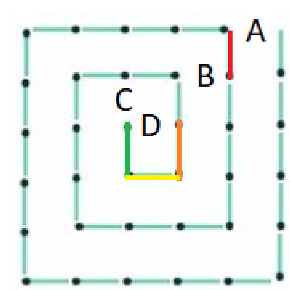
Ans 17:
Consider any two points of the same color, say white W1 and W2 and let the distance between them be 2d and W2 is to the right of W1. Consider two new points at distances 2d to the left of W1 and to the right of W2. Both of them must be black, else problem is solved. If not, then consider the midpoint of W1 and W2. It must be black else we are done. But it is also the midpoint of the two black points chosen earlier.

Ans 18:
$2^1 1 - 11^2 = 1927$

Ans 19:
Answer:



Solution:



The red line is moved to A, the orange line to B, the green line to C and the yellow line to D.

Ans 20:
Answer: 2
This single sequence in fact consists of two Series:
The first Series is 2-3-4-5-...
and
The second Series is 9-18-36-72-... .